DETERMINATION OF D_{33} AND HIGH MECHANICAL Q's USING THE ELECTRIC-LINE THEORY

A. Sauerborn

Institute for High-Frequency Techniques
University of the Bundeswehr
Munich, Germany-West

ABSTRACT

For the design of ultrasonic lithotripters certain material parameters have to be known for the operating conditions

- frequency around 25 kHz
- power up to 200 watts.

This paper deals with the determination of the mechanical quality factor of high Q materials (e.g. metals) using the electric-line theory.
Cylindric specimens have been designed the behavior of which can be calculated with simple, but accurate models. The dissipative losses are considered by integrating the space dependent distribution of the standing pressure wave. This treatment is in contrast to several papers where hyperbolic functions and T-matrices have been used.
By measuring the electrical Q_e and the impedance of two specimens with different sizes but the same materials and the same resonance frequency the mechanical Q_m and the piezo-electric charge coefficient D_{33} of the piezo-ceramic as well as the mechanical Q_m of the specimen's material can be calculated.

I. INTRODUCTION

The determination of mechanical quality factors Q_m of self resonance driven piezo-ceramic is obvious. In this case, the electric figure of merit Q_e, which can be easily measured, is identical to the mechanical quality factor. Dielectric losses can be neglected. Usually, data sheets show material constants evaluated from single discs conforming largely with the suggestions contained in the IEC publications 483 /1/. For designing transducers which are commonly driven at frequencies different from the self resonance of the piezo-discs, it is necessary to know material parameters evaluated under operating conditions. In this case, the electric quality factor Q_e of the whole transducer depends in a complicated way on geometry, mechanical Q_m's of all components, and so on. With the help of the electric-line theory, the behavior of particularly designed specimen can be completely understood, and mechanical material parameters can be extracted from electrical measurements.
It will be shown, that the mechanical quality factor Q_m and the piezo-electric charge coefficient D_{33} depend on static pressure applied during assembling the specimen (or lithotripter).
Strong nonlinear effects could be observed driving the objects with high power level.

II. ELECTRIC-LINE EQUIVALENT CIRCUIT

The Electric-Line Equivalent Circuit describes the behavior of the specimen which is driven in a single longitudinal mode. The specimen has a motionless plane at which the ceramic seems to be clamped. Therefore, a transmission-line model without any feedback consideration can be evaluated. In Fig. 1 a particular specimen with cylindrical symmetry and its corresponding electric line model are shown. With the help of this model, a mechanical reactance X_m can be calculated which has to be zero at the series-resonance frequencies. X_m is found from the electric-line theory :

$$X_m = X_1\left(1 + \frac{Z_{o,cy}\cdot\tan(\beta_{met}\cdot l_{cy})}{X_1}\right)/\left(1 - \frac{X_1\cdot\tan(\beta_{met}\cdot l_{cy})}{Z_{o,cy}}\right) - Z_{o,c}/\tan(\beta_c\cdot l_c) \quad (1)$$

$$X_1 = Z_{o,n}\cdot\tan(\beta_{met}\cdot l_n) - Z_{o,b}/\tan(\beta_{met}\cdot l_b)$$

with:

characteristic impedance : $Z_o = \sqrt{\varrho\cdot E}\cdot A$ (2)

phase constant : $\beta = \omega\cdot\sqrt{\varrho/E}$ (3)

electrical and mechanical lengths are identical

symbols:
E Young's modulus
ϱ specific gravity
A area
X reactance
l length

subscripts:
b bolt
cy cylinder
n nut
met metal
m mechanical
c ceramic

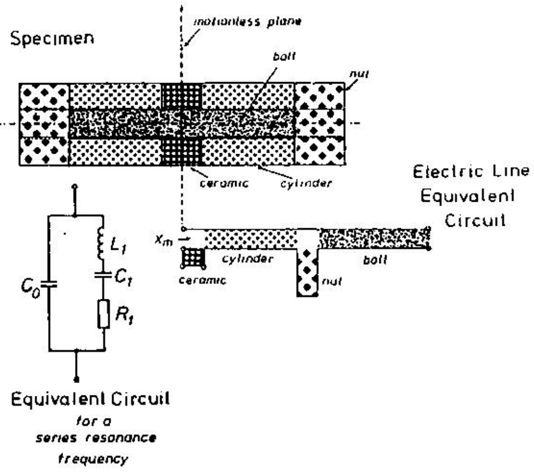

Fig. 1: Specimen and its corresponding electric-line model

III. DISSIPATIVE LOSSES

A specimen consisting of high Q materials shows standing force waves F(l) with sinusoidal distributions which cause dissipative losses.
The dissipative power P_d is:

$$P_d = \frac{\omega}{2Q_m \cdot A \cdot E} \cdot \int F^2(l)dl \qquad (4)$$

while the reactive energy W_r is:

$$W_r = \frac{1}{2 \cdot A \cdot E} \cdot \int F^2(l)dl \qquad (5)$$

F(l) can be found with the help of electric line theory owing to the fact that the force distribution along the specimen is equivalent to the voltage distribution at the corresponding electric transmission line.
Fig. 2 shows the force distribution of two specimens driven with different frequencies:

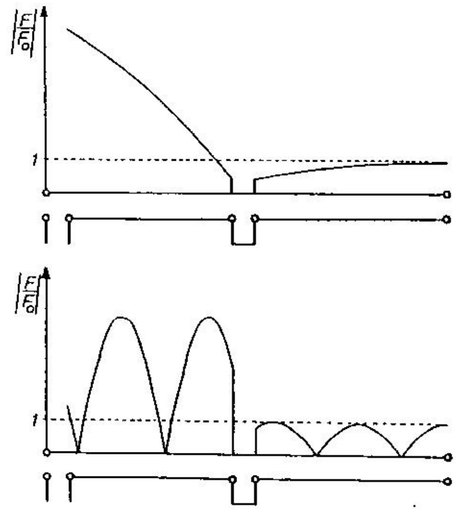

Fig. 2: Standing force waves of two specimens driven with different frequencies.

F_0 is the force at the motionless plane (see Fig.1) and depends on the driving power level P_d:

$$F_0 = 1 \text{ Newton} \cdot \sqrt{\frac{P_d}{\omega(W_{r,c}/Q_{m,c} + W_{r,met}/Q_{m,met})}} \qquad (6)$$

The values of $W_{r,c}$ and $W_{r,met}$ in (6) have to be calculated assuming a proportional force distribution with F_0 set to 1 Newton.
With the help of (6) the force at any location of the specimen can be calculated.
The electrical figure of merit is:

$$Q_e = \frac{\omega \cdot \sum W_r}{\sum P_d} = \frac{1}{R_1} \cdot \sqrt{\frac{L_1}{C_1}} \qquad (7)$$

This figure of merit can be measured very easily and will be used to determine the mechanical Q_m of the applied materials.

IV. MECHANICAL Q FACTOR

Specimen with different sizes but the same resonance frequency can be designed in finding a condition for the length of the bolt in (1) in such a way that X_m is zero.
The Q_e of both specimens ($Q_{e,1}$,$Q_{e,2}$) have to be measured and consequently the Q_m of the used materials can be calculated:

$$Q_{m,met} = \frac{Q_{e,1}\left(W_{r,met,1} - \frac{W_{r,c,1} \cdot W_{r,met,2}}{W_{r,c,2}}\right)}{W_{r,1} - \frac{Q_{e,1}}{Q_{e,2}} \cdot W_{r,c,1}\left(1 + \frac{W_{r,met,2}}{W_{r,c,2}}\right)} \qquad (8)$$

$$Q_{m,c} = \frac{Q_{e,2} \cdot W_{r,c,2}}{W_{r,2} - \frac{Q_{e,2}}{Q_{m,met}} \cdot W_{r,met,2}} \qquad (9)$$

Measurements showed that Q_e and, therefore, Q_m depend evidently on the static pressure which is produced when a specimen is assembled. This pressure was measured by integrating the resulting charge during tightening the nut.
Fig. 3 shows the dependency of $Q_{m,c}$ on the static pressure. The values of $Q_{m,c}$ differed in a wide range with different discs of the same type SONOX P8 from Ceram-Tech and ageing effects could be observed.

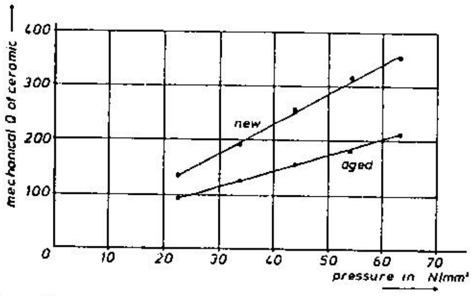

Fig. 3: Mechanical Q_m of SONOX P8 discs versus static pressure.

The $Q_{m,met}$ of the investigated metals seem to be independent of the static pressure within the interesting range. Approaching the yield point, $Q_{m,met}$ decreases.
Table 1 shows experimental results:

	ρ	E	Q_m
Steel	$7.90 \cdot 10^{-3}$	$2.09 \cdot 10^{11}$	1600
Brass	$8.38 \cdot 10^{-3}$	$9.60 \cdot 10^{10}$	2000
Aluminum	$2.81 \cdot 10^{-3}$	$7.10 \cdot 10^{11}$	3000

ρ = specific gravity [g/mm³]
E = Young's modulus [g/mm·s²]
Q_m = mechanical quality factor

Table 1: Material constants of several metals.

V. PIEZO-ELECTRIC CHARGE COEFFICIENT D_{33}

It can be shown that the motional resistance R_1 is:

$$R_1 = \frac{1}{(v\cdot\alpha_p)^2} \cdot \omega \left(\frac{W_{r,c}}{Q_{m,c}} + \frac{W_{r,met}}{Q_{m,met}}\right) \qquad (10)$$

v is the velocity at the plane between ceramic and metal as shown in Fig. 4 and

$$\alpha_p = D_{33} \cdot \frac{A_c \cdot E_c}{l_c} \qquad (11)$$

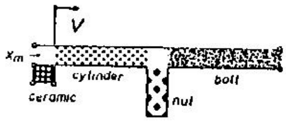

Fig. 4: Velocity v at the plane between ceramic and metal

The product of R_1 and Q_e is

$$R_1 \cdot Q_e = \frac{1}{(v\cdot\alpha_p)^2} \cdot \omega \cdot (W_{r,c} + W_{r,met}) \qquad (12)$$

Hence D_{33} is

$$D_{33} = \frac{l_c}{v\cdot A_c \cdot E_c} \cdot \sqrt{\frac{\omega(W_{r,c} + W_{r,met})}{R_1 \cdot Q_e}} \qquad (13)$$

and does not depend on mechanical Q_m's.
Fig. 5 shows values of the D_{33} evaluated as derived above.

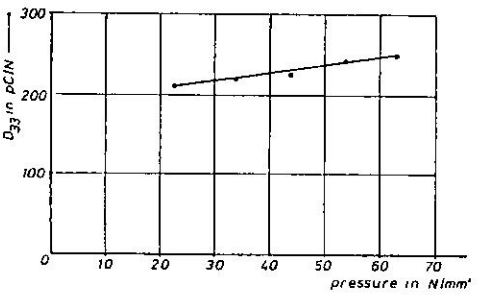

Fig. 5: Piezo-electric charge coefficient D_{33} of SONOX P8 discs versus static pressure.

A certain dependency on static pressure can be seen.

VI. NONLINEAR EFFECTS

Strong nonlinear effects could be observed driving the specimen with high power levels. A measurement system for the investigation of these effects has been developed and is shown in Fig. 6.

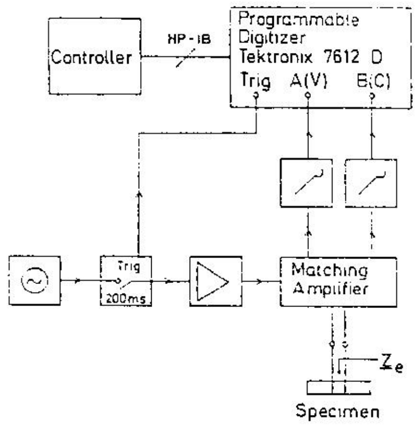

Fig. 6: Measurement system for the determination of Z_e with high level bursts.

In order to avoid thermal effects, the specimen is driven with short bursts with a duration of 200 ms only. The matching amplifier compensates the static capacitance C_0 which would cause phase errors /3/. Voltage and current of the specimen are recorded by means of a digitizer. Magnitude and phase of the impedance are computed with a least-mean square fitting method.

Fig. 7 shows measured loci of the complex impedance Z_e close to a series-resonance frequency:

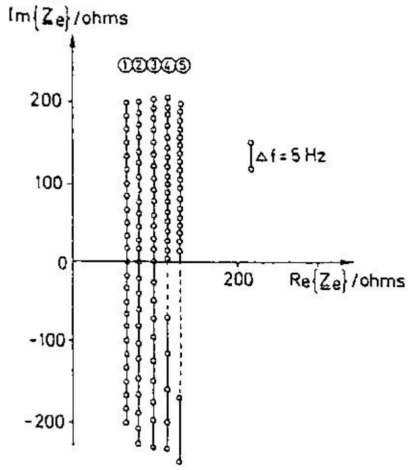

Fig. 7: Impedance loci of a specimen close to a series resonance measured with different driving levels.

The curves are measured at different driving levels of the driving voltage V.

1:V= 1 Veff	f_s= 23780 kHz
2:V= 5 Veff	f_s= 23765 kHz
3:V= 10 Veff	f_s= 23740 kHz
4:V= 30 Veff	f_s= 23610 kHz
5:V= 50 Veff	f_s= 23565 kHz

Several effects could be observed.
With increasing power

- R_1 increases
- f_s decreases
- certain phase regions are instable (dashed lines in Fig. 7)

An evaluation of the dependency of Q_e on the driving level with the help of the impedance locus is ambiguous because of the fact that the derivation of $Im(Z_e)$ shows singularities at higher power levels . Therefore, another method has to be used:
The velocity v_e at the end of the specimen's nut can be calculated by using the electric line equivalent and is proportional to the square root of the driving power P_d:

$$v_e = C\sqrt{P_d} \quad (14)$$

The constant C depends on the Q_m's of the materials the specimen is composed of. Measurements of the velocity v_e showed a strict linear relation between v_e and $\sqrt{P_d}$. Therefore, the Q_m's and, consequently, the Q_e donot depend on driving power and are linear within the investigated range. Assuming this linear behavior D_{33} depends on the driving current level and can be calculated with the help of (10) setting $Q_{m,c}$ and $Q_{m,met}$ to constant.
Fig. 8 shows values of D_{33} of the SONOX P8 discs decreasing with growing current level.

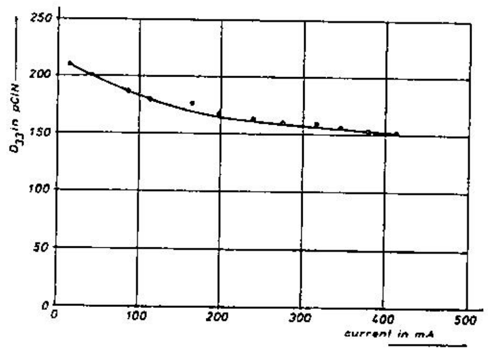

Fig. 8: D_{33} of SONOX P8 discs versus driving current.

VII. CONCLUSION

A method is presented which gives a complete understanding of the behavior of a specimen with a relative simple structure.
As a result of the investigations several material parameters have been evaluated under conditions similar to the operating conditions of more complex ultrasonic devices, e.g. ultrasonic lithotrites.
Some results are:

- Mechanical Q_m and the D_{33} of the piezo-ceramic material under investigation depend on static pressure.
- $D_{33}$of those piezo-ceramic discs decreases with increasing current.

The specimens have been designed in such a way that the power dissipated in the ceramic and in the metals are within a comparable order of magnitude. Otherwise, the greater part of the power would be dissipated in the ceramic and the influence of the $Q_{m,met}$ on the Q_e would be small and could not be evaluated with a sufficient accuracy.
Values of the mechanical Q_m of several metals are given.
For designing automatically frequency controlled power generators for more complex devices, the observed nonlinear effects must be taken into account.

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